Infinity

Exercises for Chapters 7 and 8 of Steinhart, E. (2009) *More Precisely: The Math You Need to Do Philosophy*. Broadview Press. (Version 1)

1. Explain why the idea of infinity has nothing to do with going on forever or trying to get there. Think about this carefully *before* you start writing.

2. Explain why Zeus, if he accelerates, can write down all the finite numbers in 1 second even though there is no last finite number. Again, think about this carefully *before* you start writing.

3. There are as many square numbers (1, 4, 9, 16, 25, etc.) as there are numbers. Why?

4. You love socks. On Monday you have one pair of socks for every natural number (there is a left sock and a right sock in each pair).

4.1 How is it possible to fit all these socks into a single finitely sized sock drawer in your dresser?

On Monday night, the devil destroys each one of your left socks, leaving you only with right socks. You discover this when you look at your socks on Tuesday morning.

4.2 True or false: On Tuesday, you have half as many socks as you did Monday. Explain your answer.

4.3 True or false: On Tuesday, you have exactly as many socks as you did Monday. Explain your answer.

5. You have a finitely sized urn filled with as many balls as natural numbers. Each ball has a number painted on it. You accelerate as follows: You pull out a ball in 1/2 second (this first ball has some number, but it need not be the number 1). Then you then pull out every next ball twice as fast. True or false: at 1 second, the urn is empty. Explain.

6. Use recursion to define an endless series. Give an initial rule, a successor rule, and a limit rule. To illustrate your example, feel free to use any type of object you like – angels, stars, universes, paradises, minds, weasels. Of course, you must state the relation between the objects (e.g. a successor number is greater than its predecessor; so a successor weasel is what? than its predecessor – and don't say more weasely).

7. Explain why the Diagonal Argument suggests the Power Set Argument.

8. Consider the binary tree partly shown in Figure 1. Each node in the tree (each point at which it branches) is labelled with either 0 or 1. The address of a node is the sequence of 0 or 1 branches that you take in traveling from the root on the right to the node. Thus the node whose address is 0000 is on the bottom right while the node whose address is 1111 is on the top right.

8.1 How many nodes are there in this tree? That is, what is the cardinal number of the set of nodes in the tree? Explain your answer.

8.2 How many paths are there in this tree? That is, what is the cardinal number of the set of paths in the tree? Explain your answer.

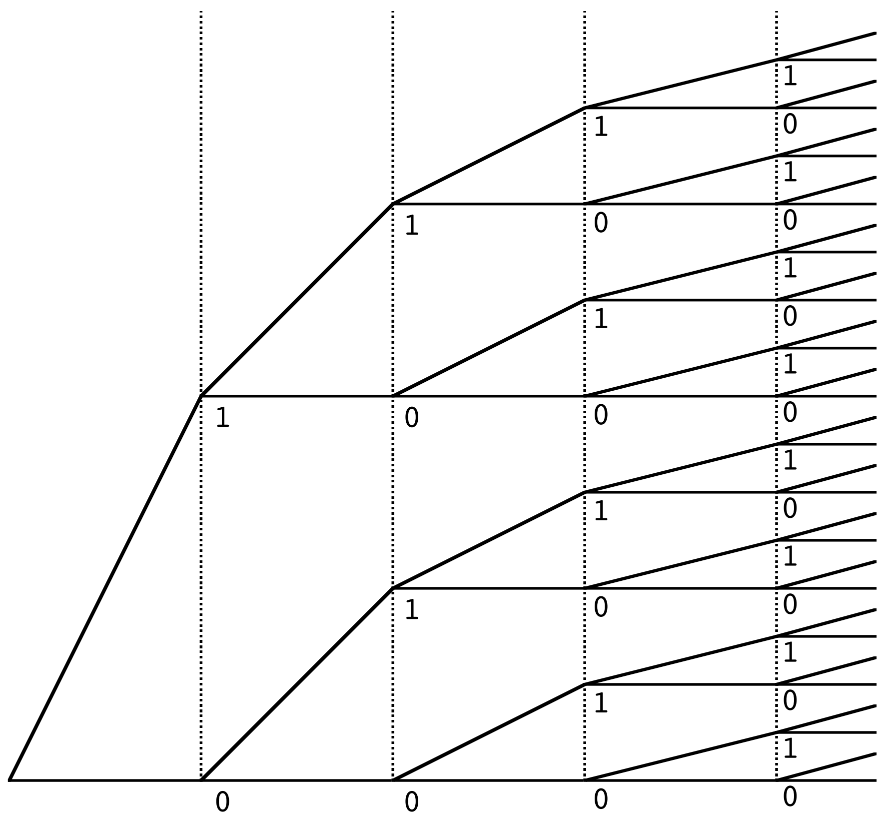

Figure 1. The first 4 iterations of an infinite binary tree.

9. Consider the collection of hereditarily finite sets. It's defined like this:

H(0) = {};

$H(n+1) = \text{pow } H(n);$

$H = \cup\{ H(n) \mid n \text{ is finite } \}.$

9.1 How many sets are there in H? That is, what is the cardinal number of H?

9.2 Is it possible to define a 1-1 correspondence between H and the natural numbers? If not, why not? If so, how would you do it?