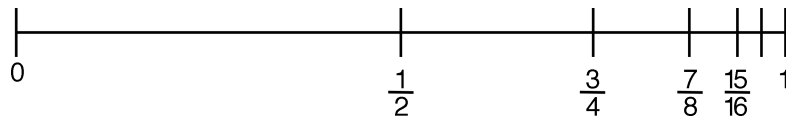


## Acceleration

### The Zeno Fractions

Infinity is an abstract pattern. When the pattern of infinity appears in time, it is acceleration; when it appears in space, it is compression.

To understand acceleration and compression, it helps to start with the Zeno Fractions. The easiest way to picture the Zeno Fractions is to picture a line that is progressively divided into smaller and smaller parts. The Zeno Fractions are the fractions under the line, they are the fractions that appear when the line is divided:



The Zeno Fractions are based on the powers of 2. For any number  $n$ , the  $n$ -th power of 2 is  $2^n$ . The first row of the table below shows some numbers; the next row shows the corresponding powers of 2. The bottom row shows the Zeno Fractions. The  $n$ -th Zeno Fraction has the form  $(2^n - 1) / 2^n$ .

Since  $(2^n - 1)$  is less than  $2^n$ , each Zeno Fraction is less than 1. As  $n$  gets bigger, the Zeno Fractions get closer and closer to 1, but they never reach it.

n	0	1	2	3	4	5	6	7	8	...	n	...
$2^n$	1	2	4	8	16	32	64	128	256	...	$2^n$	...
Zeno Fraction	0	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{31}{32}$	$\frac{63}{64}$	$\frac{127}{128}$	$\frac{255}{256}$	...	$\frac{2^n - 1}{2^n}$	...

### Acceleration: The Easter Bunny Eats Infinitely Many Jelly Beans

There is a jar filled with infinitely many jelly beans. Each jelly bean has a number  $n$  written on it. So there is a jelly bean numbered 1, 2, 3, 4, and so on for every number. Since there are infinitely many numbers, there are infinitely many jelly beans in the jar.

The Easter Bunny is going to eat some jelly beans from the jar. He eats them faster and faster. He eats each next jelly bean twice as fast as he ate the last one. The Easter Bunny *accelerates* as he eats these jelly beans.

The number on the jelly bean.	0	1	2	3	4	5	6	7	8	...
Time when the Easter Bunny eats that jelly bean.	0	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{31}{32}$	$\frac{63}{64}$	$\frac{127}{128}$	$\frac{255}{256}$	...

How many jelly beans are in the jar at time 1 minute? None. The jar is empty. The Easter Bunny ate infinitely many jelly beans in the finite time of 1 minute.

The Proof:

- (1) Assume that the Easter Bunny does not eat every jelly bean by 1 minute.
- (2) If he does not, then at 1 min, some jelly beans are left in the jar.
- (3) Pick any one of those beans. It has some number  $n$  on it.
- (4) Acceleration implies that the Easter Bunny ate the  $n$ -th jelly bean at the  $n$ -th Zeno fraction, that is, he ate it at time  $(2^n - 1) / 2^n$ .
- (5) But since  $(2^n - 1)$  is less than  $2^n$ , it follows that  $(2^n - 1) / 2^n$  is less than 1.
- (6) So the Easter Bunny ate that jelly bean at some time less than 1.
- (7) So the jelly bean you picked is not in the jar at 1 minute.
- (8) For any other jelly bean you say is in the jar at 1 minute, the same logic applies.
- (9) Therefore, there are no jelly beans in the jar at 1 minute.
- (10) Hence the assumption is false.
- (11) Therefore: The Easter Bunny has eaten every jelly bean in the jar by 1 minute.

At 1 minute, the jar is empty. Since there are infinitely many jelly beans in the jar, the Easter Bunny has eaten infinitely many jelly beans in 1 minute. By accelerating, the Easter Bunny has performed infinitely many operations in a finite amount of time.